

APPENDIX H

DERIVATION OF PRESSURE COEFFICIENTS FOR SOLUTION OF LATERAL EARTH PRESSURE PROBLEMS

H-1. Introduction.

a. Engineers are familiar with earth pressure coefficients and their use in determining pressures and forces acting on retaining walls. The most familiar and most often used are:

$$K_A = \tan^2\left(45^\circ - \frac{\Phi}{2}\right), \text{ for driving pressure}$$

$$K_P = \tan^2\left(45^\circ + \frac{\Phi}{2}\right), \text{ for resisting pressure}$$

The above represent special cases of Coulomb's equations for earth pressure coefficients.

b. These simple coefficients are proper to use only when:

- (1) Backfill surface is a level plane.
- (2) Any surcharge applied to the surface is uniform, and covers the entire surface of the backfill wedge.

c. The general equations from which these simple coefficients are obtained are subject to the following limitations:

- (1) Backfill must be cohesionless, unless the top surface is horizontal.
- (2) Fill must be completely saturated or completely unsaturated, unless the top surface is horizontal.
- (3) Top can be a constant slope, but must be an unbroken plane.
- (4) Any surcharge must be uniform and cover the entire surface of the backfill wedge.

d. The correct lateral earth force due to any backfill wedge may be obtained from the general wedge equation, and the general wedge equation is subject to none of the above limitations. This equation may be used to solve the most complicated problems of wedge geometry and surface loading.

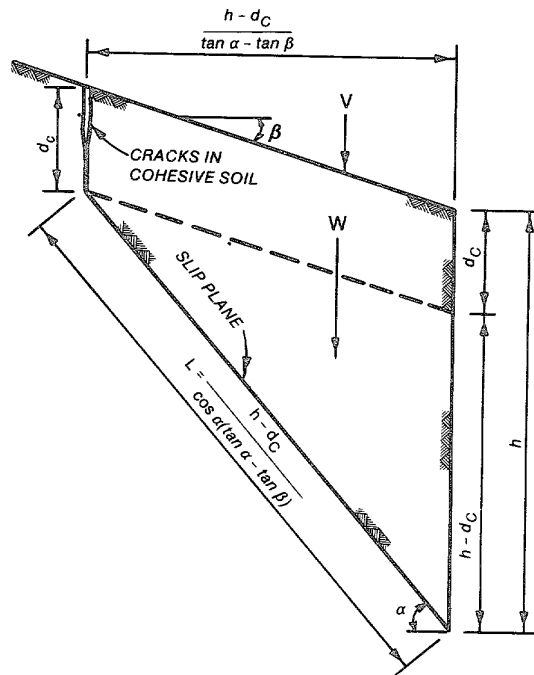
e. If lateral earth pressure coefficients are derived from the general wedge equation, these coefficients may be used in a rather simple manner to

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solve complex earth pressure problems. The general wedge equation will now be derived and the pressure coefficients obtained from the derived equation.

H-2. Driving Side Earth Pressure.

- a. A typical driving wedge is shown in the figure below.



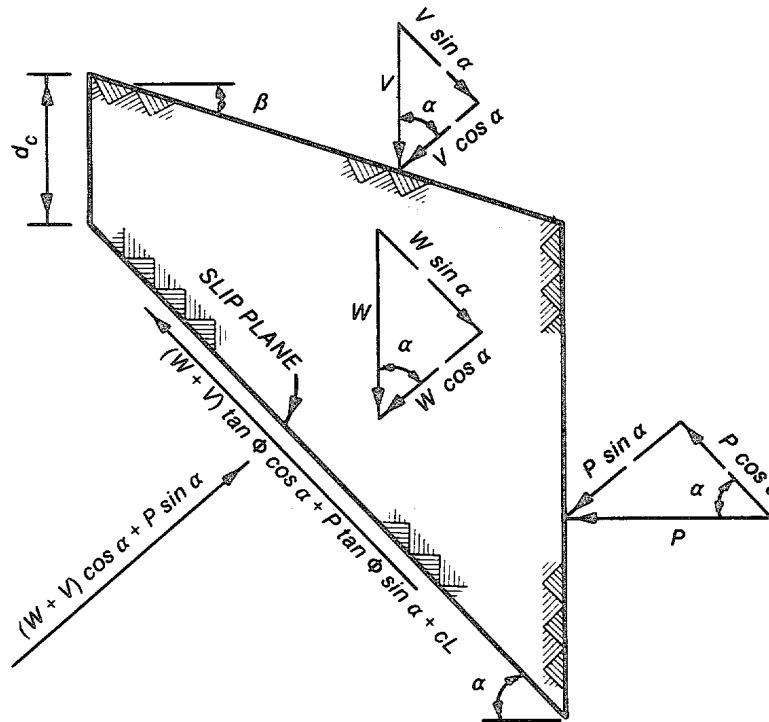
W = WEIGHT OF SOIL IN WEDGE,

V = ANY SURFACE LOAD OTHER THAN
A UNIFORM SURCHARGE

In the derivation on the following pages it will be assumed that shear on the vertical face of the wedge is zero.

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b. In the figure below, the forces acting on the wedge are shown along with their components parallel and normal to the slip plane.



Setting the summation of forces parallel to the slip plane equal to zero:

$$P (\cos \alpha + \tan \phi \sin \alpha) + (W + V) (\tan \phi \cos \alpha - \sin \alpha) + cL = 0$$

Rearranging and solving for P :

$$P = \frac{(W + V) (\sin \alpha - \tan \phi \sin \alpha) - cL}{\cos \alpha + \tan \phi \sin \alpha}$$

From the previous figure:

$$L = \frac{h - d_c}{\cos \alpha (\tan \alpha \tan \beta)} \quad \text{and} \quad W = \frac{\gamma (h^2 - d_c^2)}{2 (\tan \alpha - \tan \beta)}$$

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where

γ = unit weight of wedge material
 c = cohesion

The equation for P will now be rewritten with the individual terms for W , V , and c separated.

$$\begin{aligned}
 P = & \frac{\gamma(h^2 - d_c^2)}{2} \cdot \frac{\sin \alpha - \tan \phi \cos \alpha}{\cos \alpha + \tan \phi \sin \alpha} \cdot \frac{1}{\tan \alpha - \tan \beta} \\
 & + \frac{V(\sin \alpha - \tan \phi \cos \alpha)}{\cos \alpha + \tan \phi \sin \alpha} \\
 & - \frac{c(h - d_c)}{\cos \alpha (\tan \alpha - \tan \beta) (\cos \alpha + \tan \phi \sin \alpha)}
 \end{aligned}$$

This may be simplified and rewritten as:

$$\begin{aligned}
 P = & \frac{\gamma(h^2 - d_c^2)}{2} \cdot \frac{1 - \tan \phi \cot \alpha}{1 + \tan \phi \tan \alpha} \cdot \frac{\tan \alpha}{\tan \alpha - \tan \beta} \\
 & + \frac{V(1 - \tan \phi \cot \alpha) \tan \alpha}{1 + \tan \phi \tan \alpha} \\
 & - \frac{2c(h - d_c) \tan \alpha}{2 \sin \alpha \cos \alpha (1 + \tan \phi \tan \alpha) (\tan \alpha - \tan \beta)} \quad [H-1]
 \end{aligned}$$

c. Everything in the first term except $\gamma(h^2 - d_c^2)/2$ is the lateral earth coefficient for γ and will be called K_1 . Everything in the second term except V is the lateral coefficient for the strip surcharge and will be called K_v . Everything in the third term except $2c(h - d_c)$ is the lateral coefficient for cohesion and will be called K_c .

d. Note that K_1 and K_v both contain the term $\left(\frac{1 - \tan \phi \cot \alpha}{1 + \tan \phi \tan \alpha} \right)$.

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This is the lateral coefficient for γ when the top surface is a level unbroken plane. This term will be called K . Therefore:

$$K_1 = K \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta} \right)$$

$$K_v = K \tan \alpha$$

e. Note also that K_1 and K_c both contain the term:

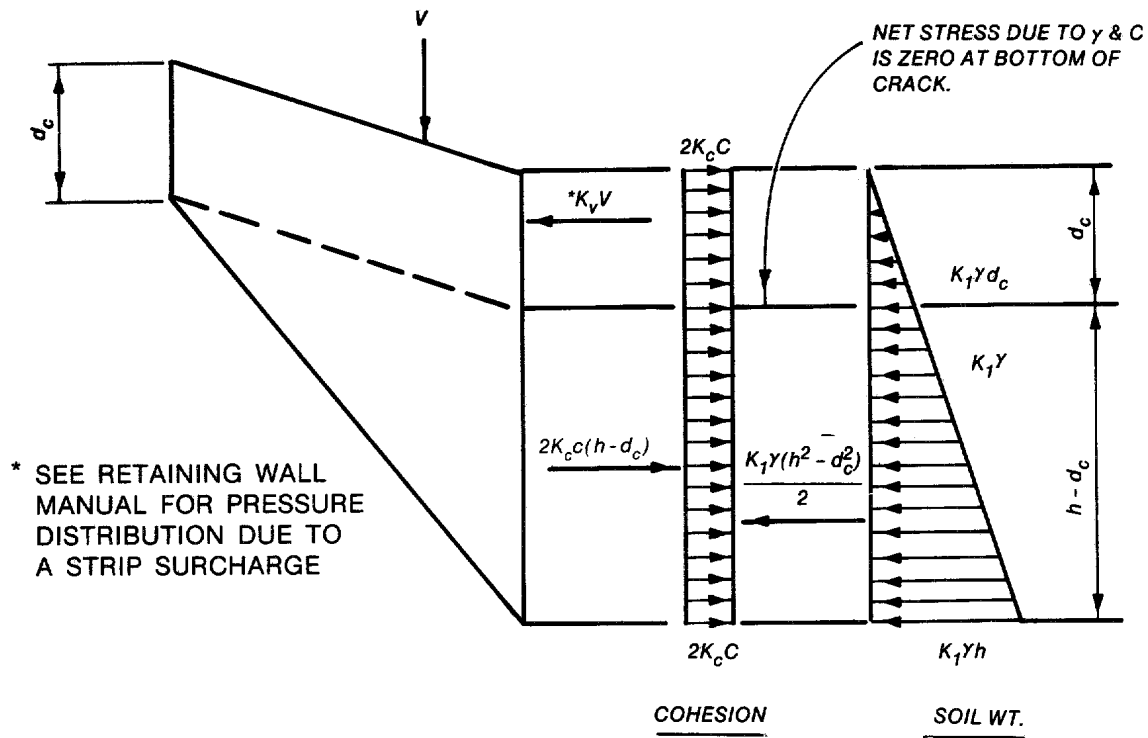
$$\frac{\tan \alpha}{\tan \alpha - \tan \beta}$$

This term which modifies K_1 and K_c for the effect of surface slope. It becomes unity when $\beta = 0$.

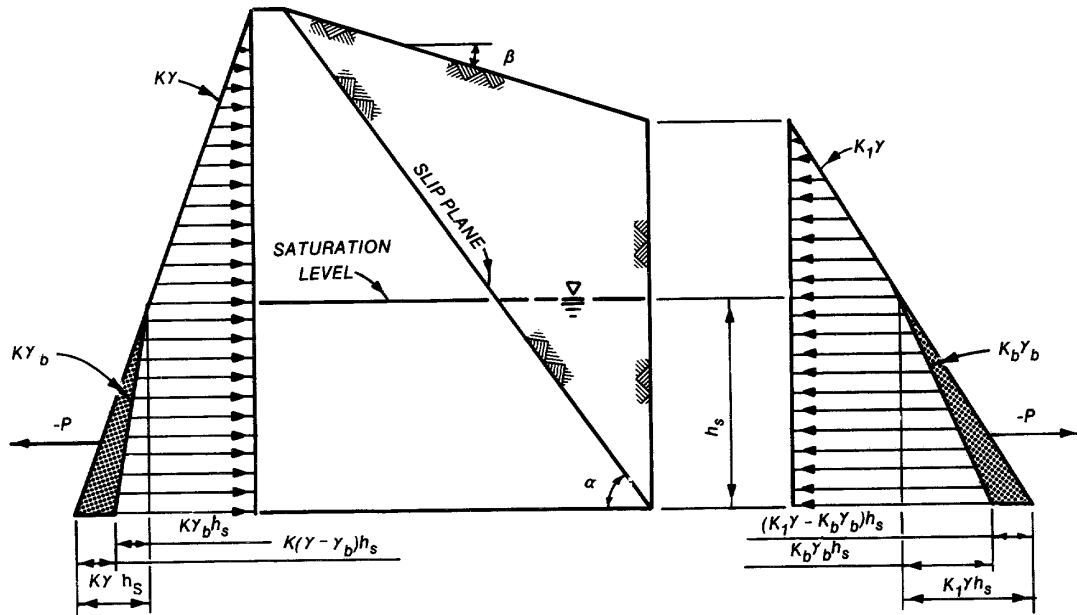
f. The equation for the total lateral force (P) produced by a driving wedge can now be written, where the wedge material possesses both cohesion and internal friction, where the top surface is a sloping plane, where the top surface supports a strip surcharge, and where the wedge is either completely saturated or completely unsaturated. The equation is:

$$P = \frac{K_1 \gamma (h^2 - d_c^2)}{2} + K_v V - 2K_c c(h - d_c) \quad [H-2]$$

g. The individual forces and pressure distributions are shown in the figure on the following page.



h. The lateral coefficient (K_b) to apply to the effective weight of fill (γ_b) below the saturation level in a partially saturated wedge will now be determined. If the water table rises in a wedge that was previously unsaturated, the lateral earth force will be reduced by the same amount on both sides of the wedge. The slope of the earth pressure diagram, acting on the vertical projection of the wedge slip plane, is $K\gamma$ above the saturation level and $K\gamma_b$ below the saturation level. The slope of the earth pressure diagram, acting on the vertical face of the wedge, is $K_1\gamma$ above the saturation level and $K_b\gamma_b$ below the saturation level. See the figure on the following page.



The force reduction on each side of the wedge (-P) is represented by the shaded parts of the pressure diagrams.

$$\frac{K(\gamma - \gamma_b)h_s^2}{2} = \frac{(K_1\gamma - K_b\gamma_b)h_s^2}{2}, \quad K_b\gamma_b = K_1\gamma - K\gamma + K\gamma_b$$

$$K_b = (K_1 - K)\left(\frac{\gamma}{\gamma_b}\right) + K$$

$$K_b = K \left[1 + \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta} - 1 \right) \left(\frac{\gamma}{\gamma_b} \right) \right]$$

i. Summarizing, the coefficients for driving wedges are:

$$K = \frac{1 - \tan \phi \cot \alpha}{1 + \tan \phi \tan \alpha}, \quad \text{basic value}$$

$$K_1 = K \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta} \right), \quad \text{apply to moist fill above saturation level}$$

$$K_b = K \left[1 + \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta} - 1 \right) \left(\frac{\gamma}{\gamma_b} \right) \right], \text{ apply to buoyant weight of fill}$$

below the saturation level.

$K = K \tan \alpha$, multiply times V to obtain lateral force due to strip
surcharge

$$K_c = \frac{1}{2 \sin \alpha \cos \alpha (1 + \tan \phi \tan \alpha)} \cdot \frac{\tan \alpha}{\tan \alpha - \tan \beta}, \text{ multiply times } 2c$$

to obtain negative pressure due to cohesion.

j. A problem remains that must be solved before the coefficients can be used for calculating driving pressures and forces, and that problem is the value of the critical slip-plane angle. The critical angle is the angle which produces the maximum driving force. The equations for calculating this angle are derived in Appendix G and presented in Chapter 3 of the manual, and are as follows:

$$\alpha = \tan^{-1} \left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2} \right) \quad [3-25]$$

where

$$c_1 = \left[2 \tan^2 \phi_d + \frac{4c_d(\tan \phi_d + \tan \beta)}{\gamma(h + d_c)} - \frac{4V \tan \beta (1 + \tan^2 \phi_d)}{\gamma(h^2 - d_c^2)} \right] \div A \quad \text{Equation 3-28}$$

$$c_2 = \left[\tan \phi_d (1 - \tan \phi_d \tan \beta) - \tan \beta + \frac{2c_d(1 - \tan \phi_d \tan \beta)}{\gamma(h + d_c)} + \frac{2V \tan^2(\beta (1 + \tan^2 \phi_d))}{\gamma(h^2 - d_c^2)} \right] \div A \quad \text{Equation 3-29}$$

and

$$A = \tan \phi_d + \frac{2c_d(1 - \tan \phi_d \tan \beta)}{\gamma(h + d_c)} - \frac{2V(1 + \tan^2 \phi_d)}{\gamma(h^2 - d_c^2)} \quad [3-30]$$

k. The effect of water is accounted for by using the average unit weight of soil for γ in the above equations. The average unit weight is based on the moist unit weight of soil above the water table and the buoyant unit weight below the water table. The effect of seepage should be considered in determining the buoyant weight.

H-3. Resisting Side Earth Pressure.

a. Pressure Coefficients. The pressure coefficients for the resisting side are

$$K = \frac{1 + \tan \phi \cot \alpha}{1 - \tan \phi \tan \alpha}, \quad K_1 = K \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta} \right)$$

$$K_v = K \tan \alpha$$

$$K_b = K \left[1 + \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta} - 1 \right) \left(\frac{\gamma}{\gamma_b} \right) \right]$$

$$K_c = \frac{1}{2 \sin \alpha \cos \alpha (1 - \tan \phi \tan \alpha)} \cdot \frac{\tan \alpha}{\tan \alpha - \tan \beta}$$

The term for cohesion ($2K_c c$) is positive for resisting pressure calculations, not negative as it was for driving pressure. Therefore, the crack of depth d_c does not exist on the resisting side.

b. Critical Slip-Plane Angle. The equation for the critical slip-plane angle for a resisting side wedge is

$$\alpha = \tan^{-1} \left(\frac{-C_1 + \sqrt{C_1^2 + 4C_2}}{2} \right)$$

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Values for c_1 and c_2 are determined from the equations in Appendix G.

c. With the above changes, the method for determining resisting pressures and forces is the same as for the driving case.